MAC2311 Exam 2 Review

1.) Take the derivatives of the following functions:

(a.) $f(x) = \dfrac{x^2 - x + 1}{(x-1)^2}$

(b.) $g(x) = e^{-x} \ln(x^2 + 2)$

(c.) $h(x) = \begin{Bmatrix} \dfrac{x^2 - 1}{x+2}; x > -1 \\ x + 2; x \le -1 \end{Bmatrix}$

(d.) $k(x) = x^{\sin(x)}$ (Hint: Use logarithmic Differentiation)

2.) Use Differentiation Rules to compute the following:

(a.) Let $h(x) = [f(x) + 2][g(x) + x]$. Find $h'(1)$ if $f(1) = 2, f'(1) = -1, g(1) = 0,$ and $g'(1) = 4$.

(b.) Let $h(x) = f(g(x) + 2)$. Find $h'(0)$ if $g(0) = 2, g'(0) = -1, f(0) = 4, f'(0) = 6, f'(4) = 3,$ and $f(4) = -2$.

3.) Let $g(x) = \sin^2(\cos(4x))$. Find $g''(x)$.

4.) It is known that there exists an implicit relationship between x and y such that:

$$5x^2 y - y^3 = 1 + x^2.$$

(a.) Find $\dfrac{dy}{dx}$.

(b.) Find the equation of the tangent line to the curve at the point (1,2).

5.) Evaluate the following limit: $\lim_{x\to 0} \dfrac{\sin^2(8x)}{\tan^2(x)}$.

6.) Compute the following limits using L'Hopspital's Rule:

(a.) $\lim_{x\to\infty} \dfrac{e^x}{x^2}$

(b.) $\lim_{x\to\infty} x^{\frac{1}{x}}$

(c.) $\lim_{x\to 0} \dfrac{e^x - 1 - x}{x^2}$

7.) Find the horizontal asymptote(s), if any, of $f(x) = e^{-2x^2}$.

8.) Find a and b so that $g(x)$ is continuous and differentiable for all values of x

Where $g(x) = \begin{Bmatrix} ax + 4, x < 1 \\ 3x^2 + b, x \geq 1 \end{Bmatrix}$.

9.) Find the derivatives of the following two functions using the limit definition of the derivative:

(a.) $f(x) = \dfrac{x+2}{x}$

(b.) $h(x) = \sin(x)$

10.) The position, in meters, of a particle at time t is given by

$$f(t) = t^3 - 9t^2 + 15t - 12,$$

where t is measured in seconds.

(a.) Find the velocity function, $v(t)$.
(b.) At what time(s) is the particle at rest?
(c.) When is the particle moving in the positive direction?
(d.) What is the displacement of the particle from $t = 0$ to $t = 8$.

(e.) What is the particle's average velocity from $t=2$ to $t=4$?
(f.) What is the particle's instantaneous velocity at $t=3$ seconds?
(g.) If $f(t)$ instead represented the temperature of an object after being immersed in a reactor for t seconds, interpret the instantaneous rate of change after one second.

11.) Find the slope of the <u>normal</u> line to $f(x)=\tan^{-1}(3x^2)$ at $x=1$.

12.) Find the horizontal asymptotes of the following functions:

(a.) $y=\dfrac{2e^x}{e^x-5}$

(b.) $y=\dfrac{\sqrt{4x^2+1}}{x+1}$

13.) Find derivatives of the following functions, and simplify the results:

(a.) $f(x)=5^{\tan^2(x)}$

(b.) $g(x)=a^{2x}+x^{e^2}-e^{3x+1}+e^{4a}$

14.) <u>OPTIONAL</u>: An airplane 2500 feet above the ground is flying away from an observer on the ground at a rate of 300 miles per hour. If the plane is moving parallel to the ground, at what rate is the distance between the observer and the plane increasing when the plane is 3125 feet from away from the observer? (HINT: Distance = $\sqrt{x^2+2500^2}$, and $x=300t$; This is a chain rule problem)