# MAC 2233
# Exam 2 Review

**1.)** Given the following function:

$$f(x) = 3x - x^2$$

**a)** Use the definition of the derivative to find the slope of the tangent line at the point (1,2).

**b)** Write an equation of the tangent line to the function that passes through the same point.

**2.)** The demand function for racquetball equipment sold by a given store is given by

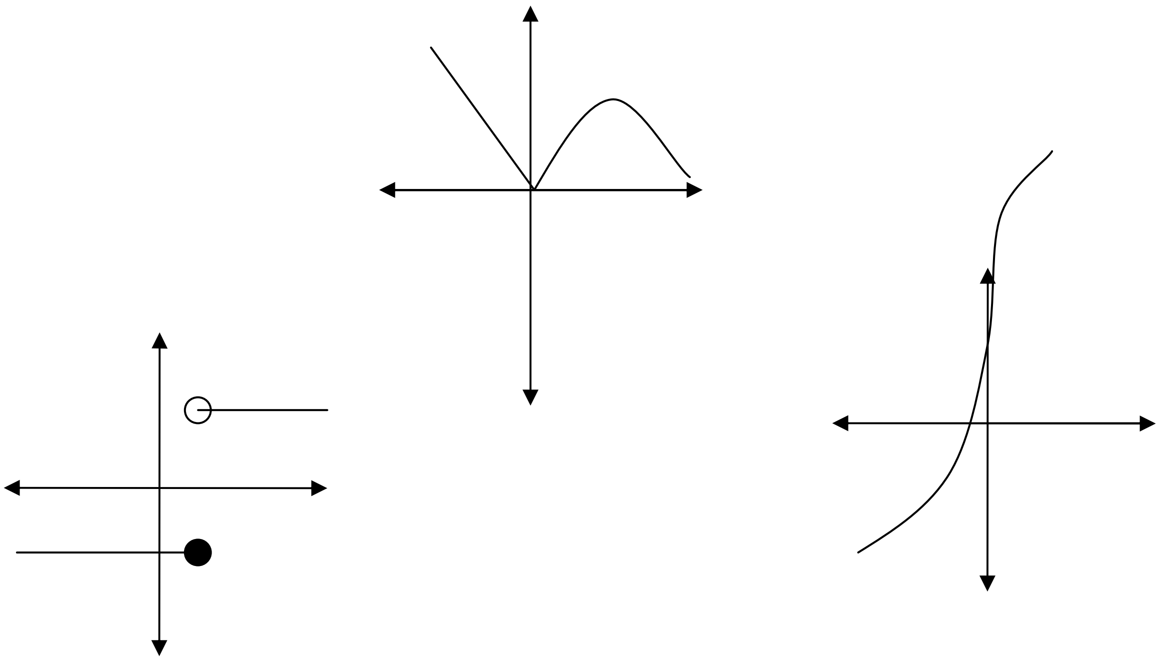
$$p = f(x) = -0.1x^2 - x + 40$$

where $p$ is measured in dollars, and $x$ is measured in units of a thousand.

**a)** Find the average rate of change in the unit price of a piece of equipment if the quantity demanded is between 5000 ($x = 5$) and 6000 ($x = 6$) pieces of equipment.

**b)** What is the instantaneous rate of change of the unit price if the quantity demanded is 5000 $(x = 5)$?

**3.)** Which of the functions graphed below are <u>not</u> differentiable?



**4.)** Differentiate the following functions:

**a)** $g(x) = \dfrac{x^3 + 2x^2 + x - 1}{\sqrt{x}}$

**b)** $f(x) = (3x^2 - 1)(x^2 - \dfrac{1}{x})$

**c)** $h^{'}(1)$ if $h(x) = \dfrac{f(x)g(x)}{f(x) - g(x)}$ and

$f(1) = 2, f^{'}(1) = -1, g(1) = -2, g^{'}(1) = 3$

**5.)** Let $p(x) = -0.3x + 20$ and $C(x) = 5x + 12$ be the demand and cost functions, respectively.

**a)** Find the profit function, $P(x)$.

**b)** Find the <u>marginal</u> profit when production is 20 units.

**c)** Interpret your answer from part (b.), and use it to estimate the profit from selling 21 units.

**d)** Find the intervals on which profit is increasing.

**6.)** Use differentiation rules to compute $h'(1)$, where

$$h(x) = f(g(x) + x^2) \text{, and given that}$$

$$g(1) = 2, g'(0) = -1, f'(3) = 4, f(-1) = 0, g'(1) = 2.$$

**7.)** The quantity demanded each week, $x$ (in units of a hundred), of the Mikado miniature camera is related to the unit price, $p$ (in dollars), by the demand equation:

$$x = \sqrt{400 - 5p}$$

**a)** Is the demand elastic or inelastic when $p = 40$? When $p = 60$?

**b)** When is the demand unitary?

**c)** If the unit price is lowered slightly from \$60, will the revenue increase or decrease?

**8.)** Write the equations of the horizontal tangent lines of

$$f(x) = (2x-1)^3(3x-1)^4.$$

**9.)** A ball is thrown up off of a rooftop 320 feet above the ground with an initial velocity of 128 ft/sec.

**a)** Using the generic formula for the height of a ball (modeled by a parabolic path): $h(t) = -16t^2 + v_0 t + h_0$, where $v_0$ is the ball's initial velocity, and $h_0$ is the ball's initial height, find the equation describing the height of the ball at every time, $t$.

**b)** Find the velocity, $v(t)$, and acceleration, $a(t)$, functions.

**c)** What is the velocity of the ball just before it hits the ground?

***d)*** Find the average velocity on the interval [1,3].

***10.)*** Find $a$ and $b$ so that $g(x)$ is continuous and differentiable for all values of $x$, where

$$g(x) = \begin{cases} ax+4 & \text{if } x<1 \\ 3x^2+b & x \geq 1 \end{cases}$$

***11.)*** Find the values of $a$ so that the tangent line to $y = x^2 - 2\sqrt{x} + 1$ is perpendicular to $ay + 2x = 2$ at $x = 4$.

***12.)*** A business is currently selling 1000 novelty wind chimes for $100 each. It is estimated that for each decrease in price of $20, the monthly sales will increase by 200 chimes.

***a)*** Find the demand function, $p(x)$, where $p(x)$ is measured in dollars and $x$ is the number of chimes sold.

***b)*** Suppose a company is increasing the production of chimes by 15 chimes per month. Find the rate at which revenue is changing with respect to time when production is at 500 chimes. Include units.

***13.)*** Find an equation of the tangent line to the graph of the following function at the point (1,1):

$$h(x) = x^2y^3 - y^2 + xy - 1 = 0$$

***14.)*** The relationship between Cunningham Realty's quarterly profits, $P(x)$, and the amount of money $x$ spent on advertising per quarter is described by the function

$$P(x) = -\frac{1}{8}x^2 + 7x + 30$$

where both $x$ and $P(x)$ are both measured in units of a thousand. Use differentials to approximate the increase in profits when advertising expenditure each quarter is increased from $24,000 to $26,000.