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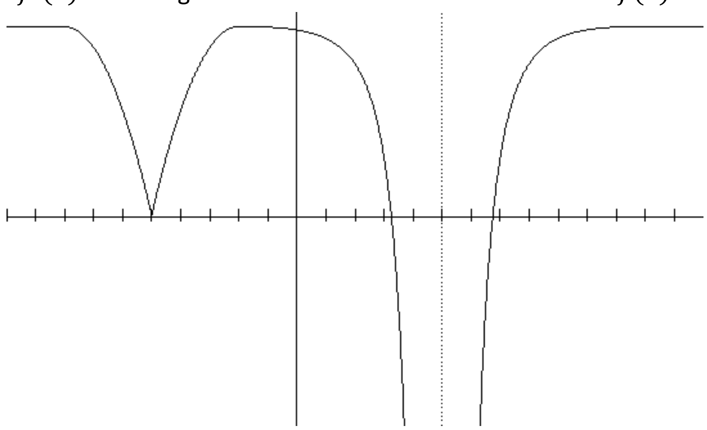
1) How many local extrema does the function $f(x) = \frac{e^x}{1-x+x^2}$ have, and are they local minima, maxima, or neither?

2) Find the absolute minimum and maximum values for each of the following functions.
   a) $f(x) = \ln(x^2 + x + 1)$ over the interval $[-1,1]$
   b) $f(t) = t\sqrt{4 - t^2}$ over the interval $[-1,2]$

3) Consider the graph of $f'(x)$ which is given below for the continuous function $f(x)$.



   a) Where does $f(x)$ have critical numbers?
   b) On what intervals is $f(x)$ increasing/decreasing?
   c) On what intervals is $f(x)$ concave up/down?
   d) Sketch the graph of $f(x)$ showing the relative extrema and points of inflection using the answers from parts (a) through (c).

4) A young gentleman is standing 10 feet from a building where his significant other is throwing his stuff out of their apartment window 20 feet above the main apartment entrance which is horizontally lined up with the gentleman's line of site if he looks strait ahead. However, the gentleman can't take his eyes off of the falling objects… especially the T.V. … and his line of sight makes an angle, θ, relative to him looking at the apartment entrance. How fast is this angle decreasing when his T.V. is half-way down assuming its falling at a rate of 30 ft/second at that position?

5) Find the critical numbers of the following functions. Use the second derivative test to determine which critical numbers represent local maximums and which represent local minimums.
   a) $f(x) = x^2 e^{-3x}$
   b) $g(x) = x^{-2}\ln(x)$
   c) $k(\theta) = 2\cos(\theta) + \sin^2(\theta)$

6) Approximate 1.97^6 using differentials.

7) Consider the following functions,

$$f(x) = \frac{x^2}{(x-2)^2} \quad , \quad g(x) = x^{\frac{2}{3}} + 2x^{\frac{1}{3}}$$

Sketch a graph of the following function and list the locations

a) The vertical and horizontal asymptotes
b) Intervals where the function is increasing, decreasing, concave up, and concave down.
c) Local Minimum, local Maximum, and Points of Inflection

8) Find the limit. You may use L'Hospital's Rule if it applies. If it does not apply state why.

a) $\lim_{x\to\infty} \frac{e^x}{x^2}$
b) $\lim_{x\to 0} \frac{x+\sin x}{x+\cos x}$
c) $\lim_{x\to\infty} x^{\frac{1}{x}}$
d) $\lim_{x\to\infty} \frac{e^x-1-x}{x^2}$
e) $\lim_{x\to\pi/4}(1-\tan x)\sec x$
f) $\lim_{x\to 0^+}(\sin x)^{\tan x}$

9) How many inflection points does the function have over the entire real number line?

$$g(x) = 3x^5 - 10x^4 + 10x^3$$

10) According to Hooke's Law, the force exerted on a spring of mass, m, is given by the following equation

$$F = -kx$$

where k is the spring constant and x is the distance that the spring is extended from its resting position. Determine the velocity of the spring when the rate of change of the force with respect to time is 10 Newtons and the spring constant is 3.

11) Gravel is being dumped from a conveyor belt at a rate of 30 ft^3/min. As it falls from the conveyor belt it forms a pile in the shape of a cone whose base diameter and height are always equal. How fast is the height of the pile increasing when the pile is 10 ft high?

12) What is the linearization $L(x)$ of the function $\sqrt[3]{x}$ at $a = 8$?

13) The radius of a circular sphere is given as 24 cm with a maximum error in measurement of 0.2 cm.
a) Use differentials to estimate the maximum error in the calculated volume of the disk.
b) What is the relative error? What is the percentage error?

14) Verify the function satisfies the hypothesis of the Mean Value Theorem on the interval $[0,2]$. Then find all the numbers, c, that satisfy the conclusion of the Mean Value Theorem.

$$f(x) = x^3 + x - 1$$

15) Verify the function satisfies the hypothesis of Rolle's Theorem on the interval $[0,2\pi]$. Then final all the numbers, c, that satisfy the conclusion of Rolle's Theorem.

$$\sin x + \cos x$$

16) A water trough is 10 m long and a cross-section has the shape of an isosceles trapezoid that is 0.3 m wide at the bottom, 0.8 m wide at the top, and has a height 0.5 m. If the trough is being filled with water at the rate of 0.2 m^3/min, how fast is the water level rising when the water is 0.3 m deep?