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- 1) Use the limit definition of the derivative to explore why  $|x|$  is not differentiable at the point  $(0,0)$ .

- 2) Differentiate the following functions:

a)  $f(x) = \frac{x^3 + 2x^2 + x - 1}{\sqrt{x}}$

b)  $g(x) = (3x^2 - 1)(x^2 - \frac{1}{x})$

- 3) Differentiate the following functions:

a)  $h(x) = \frac{x^2 - x + 1}{(x-1)^{2/3}}$

c)  $m(x) = \ln(\ln(e^x + 1))$

d)  $n(x) = \sin^2(\cos(4x))$

b)  $k(x) = \begin{cases} -\frac{x^2-1}{x+2}, & x > -1 \\ x+2, & x \leq -1 \end{cases}$

e)  $q(x) = a^{2x} + x^{e^2} - e^{3x+1} + e^{4a}$ ,  
a is constant

f)  $p(x) = 5^{\tan^2(x)}$

- 4) Find...

$$\lim_{h \rightarrow 0} \frac{\sqrt[5]{x+h} - \sqrt[5]{x}}{h}$$

- 5) Take the derivatives of the following functions:

a)  $f(x) = 5^{\tan^2(x)}$

c)  $h(x) = \frac{e^{3x+1}(x^2+3)^3}{\sqrt{2x-1}}$

b)  $g(x) = x^{\sin(x)}$

- 6) Given  $f(x) = 3x^2 \sqrt[3]{4-x^2}$

a) Find  $\frac{df}{dx}$

- b) Where are the horizontal and vertical tangents?

- 7) Find  $a$  and  $b$  so that  $g(x)$  is continuous and differentiable for all values of  $x$ , where

$$g(x) = \begin{cases} ax + 4, & x < 1 \\ 3x^2 + b, & x \geq 1 \end{cases}$$

- 8) It is known that there exists an implicit relationship between the variables  $x$  and  $y$  satisfying the relation below:

$$5x^2y - y^3 = 1 + x^2$$

a) Find  $\frac{dy}{dx}$ .

- b) Find the equation of the tangent line to the curve at the point  $(1,2)$ .



- 9) The functions  $h(x)$  and  $k(x)$  below are combinations of the differentiable functions  $f(x)$  and  $g(x)$ .

$$h(x) = \frac{f(x)g(x)}{f(x) - g(x)}, \quad k(x) = f^2(g(x) + x^2)$$

- a) Find  $h'(x)$ .  
b) Find  $k'(x)$ .

The following table lists some of the values of functions  $f(x)$ ,  $f'(x)$ ,  $g(x)$  and  $g'(x)$ .

	$x = 1,$	$x = 0,$	$x = -1,$
$f(x)$	2	0	-1
$f'(x)$	-1	2	4
$g(x)$	-2	1	-3
$g'(x)$	3	-1	2

- c) Find  $h'(1)$   
d) Find  $k'(1)$

- 10) Evaluate the following limits:

a)  $\lim_{x \rightarrow 0} \frac{\sin^2(8x)}{\tan^2(x)}$

d)  $\lim_{x \rightarrow 1} \frac{\sin(x-1)}{x^2+x-2}$

b)  $\lim_{\theta \rightarrow \pi} \frac{\sin \theta}{\theta + \tan \theta}$

e)  $\lim_{x \rightarrow \infty} \frac{\cos(x+1)}{x^2-2x-3}$

c)  $\lim_{\theta \rightarrow 0} \frac{\sin \theta}{\theta + \tan \theta}$

- 11) Derive the derivatives of the following functions (by either using the limit definition or Implicit Differentiation).

a)  $f(x) = \frac{x+2}{x}$

c)  $h(x) = \tan^{-1}(x)$

b)  $g(x) = \sin(x)$

- 12) Let  $f(x) = \tan^{-1}(3x^2)$

- a) Find the slope of the normal line to at  $x = 1$ .  
b) Find  $f''(0)$ .

- 13) For what values of  $x$  does the function  $g(x) = x + 2 \sin x$  have horizontal tangent lines?

- 14) Suppose  $f(x) = ax^2 + bx + c$  and that the tangent lines at  $x = 1$  and  $x = -1$  have slopes  $-8$  and  $-1$  respectively, and that the point  $(2,15)$  is a point on the graph. What are the values of  $a$ ,  $b$ , and  $c$ ?

- 15) Find the values of  $a$  so that the tangent line to  $y = x^2 - 2\sqrt{x} + 1$  is perpendicular to the line  $ay + 2x = 2$  at  $x = 4$ .



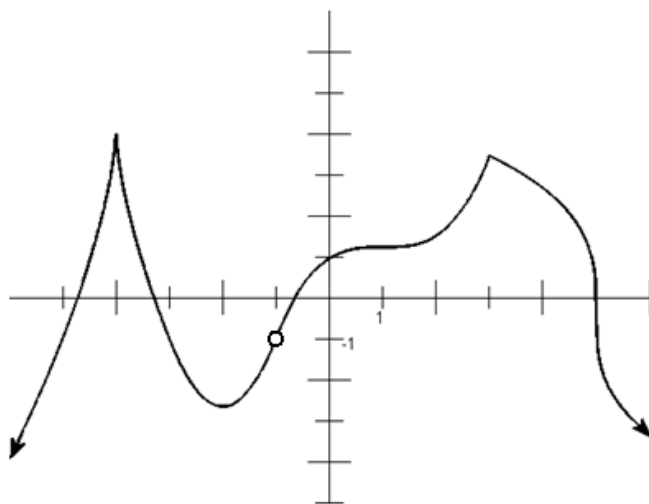
16) The position, in meters, of a particle at time  $t$  is given by

$$f(t) = t^3 - 9t^2 + 15t - 12$$

where  $t$  is measured in seconds.

- a) What is the displacement from 0 to 8 seconds.
- b) Find the particles average rate of change from 2 to 4 seconds.
- c) Find a function for the velocity at time  $t$ .
- d) What is the particle's instantaneous rate of change at 2 seconds.
- e) At which time(s) is the particle at rest?
- f) When is the particle moving in the positive direction?
- g) Find a function for the acceleration at time  $t$ .

17) Given the graph below:



Identify where the function...

- a) ... is increasing,
- b) ... is decreasing,
- c) ... has horizontal tangents,
- d) ...has vertical tangents,
- e) ... Is not differentiable. Can you classify the types of non-differentiability.
- f) Now use the answers above to sketch a graph of the derivative.