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1) Find all solutions to the following equation on the interval $[-\pi, \pi]$,
$$2\sin^2(\theta) + 3\sin(\theta) = -1$$

2) Given that the functions,
$$k(x) = \sqrt{1-x^2}, \text{ on } [0,1]$$
$$f(x) = e^{3x+1}$$
   a) Find $k^{-1}(x)$.
   b) What is the domain and range of $k^{-1}(x)$?
   c) Find $f^{-1}(x)$.
   d) What is the domain and range of $f^{-1}(x)$?

3) Write the function $g(x)$ if $g(x)$ is the graph of $f(x) = (x-1)^3 - 5$ reflected across the y-axis, reflected across the x-axis, then shifted right 3 units and down 1 unit.

4) Solve the following equation for the variable x: $\ln(ln(x^2)) = 0$

5) Determine which of the following functions have vertical asymptotes, and if so, at what x-values do they occur at? What are the equations for the horizontal asymptotes?
   a) $\frac{x+1}{x^2-4}$
   b) $\tan\left(x + \frac{\pi}{4}\right)$
   c) $\frac{2e^x}{e^x-1}$
   d) $1 + \ln(x^2)$

6) Determine if the following functions are even, odd, or neither:
   a) $f(x) = x + \tan(x)$
   b) $h(x) = e^x + e^{-x}$

7) An oil spill expands in a circle such that the radius of the spill quadruples every 2 days. If the initial radius of the spill was 10 meters:
   a) Find a formula for the radius of the spill at all times t, where t is measured in days.
   b) Find the radius of the spill after 4 days.
   c) When will the radius be 25 meters? Leave your answer in pure form, that is, no decimals.

8) It is known that for a particular cone-shaped object, its diameter and height are always equal. Find an equation for the volume of this object in terms of its radius.

9) Find the domain of each function below:

a)

$$g(x) = \frac{|x|}{x^2}$$

b)

$$a(x) = \begin{cases} x+9 & if \quad x < -3 \\ -2x & if \quad |x| \leq 3 \\ -6 & if \quad x > 3 \end{cases}$$

c) $(f\ o\ g)(x)$ where $f(x) = \frac{1}{\sqrt{x-1}}$ and $g(x) = \frac{2}{x^2}$.

d) $f(x) = \ln\left(\frac{x-3}{x+2}\right)$

10) Consider the function,

$$g(x) = \frac{|x-3|}{x^2 - 6x + 9}$$

a) Write the function in its piece-wise defined form.
b) What is the domain of the function
c) Sketch a graph of this function.
d) Does the limit as $x \to 3$ exist for this function? Explain why or why not.

11) A projectile is thrown from the ground with an initial velocity of 25 ft/s and an acceleration of 10 ft/s^2 and thus its height, $h(t)$, after t seconds is given by:

$$h(t) = -10t^2 + 25t$$

a) Find the average velocity from the time interval $[2,4]$.
b) Find the average velocity from the time interval $[2,2+h]$.
c) Use limit and part (b) to find the instantaneous velocity after 2 seconds.
d) What would be the equation of the tangent line to $h(t)$ at $t = 2$?

12) Find the value of a for which the following piecewise function will be continuous at $x = 1$.

$$f(x) = \begin{cases} \dfrac{|x-1|}{x^2-1} & if \quad x < 1 \\ ax^2 + 6 & if \quad x \geq 1 \end{cases}$$

13) Use the squeeze theorem to prove that the following limit approaches zero.

$$\lim_{x \to 0} |x| \sin\left(\frac{2}{x}\right)$$

14) Explain why the intermediate value theorem does or does not guarantee a solution of the given equation in the specified interval:

a) $(x-1)^4 + x - 4 = 0$ on the interval $(2,3)$
b) $\tan(x) = 0$ on the interval $\left(\frac{\pi}{4}, \frac{3\pi}{4}\right)$
c) $\sqrt[3]{x-2} = 3 - x$ on the interval $(-2,-1)$
d) $\ln(x) = e^{-x}$ on the interval $(1,2)$

15) Let

$$f(x) = \begin{cases} x^2 - 9 & if \quad x < -3 \\ x + 3 & if \quad -3 \le x < 1 \\ -\dfrac{x^2 - 2x - 8}{x^2 - 7x + 12} & if \quad x \ge 1 \end{cases}$$

a) Sketch $f(x)$.

b) Find the following limits:

$$\lim_{x \to 1} f(x) \quad and \quad \lim_{x \to 3} f(x)$$

c) Find and describe any discontinuities of $f(x)$.

d) If possible, make $f(x)$ continuous at any of its points of discontinuity.

16) Complete the following actions given the following function.

$$f(x) = \frac{x^2 - x - 2}{|2 - x|}$$

a) Find $\lim_{x \to 2^+} f(x)$.

b) Find $\lim_{x \to 2^-} f(x)$.

c) Find $\lim_{x \to 2} f(x)$.

d) Sketch the graph of $f(x)$.

17) Use algebra and/or other simplification techniques to evaluate the following limits:

a)

$$\lim_{x \to 2^-} \frac{x^2 + 2x - 8}{x^2 - 2x}$$

b)

$$\lim_{x \to 0^+} \frac{x^2 + 2x - 8}{x^2 - 2x}$$

c)

$$\lim_{x \to 3} \frac{\sqrt{x} - \sqrt{3}}{x^2 - 9}$$

d)

$$\lim_{h \to 0} \frac{\dfrac{1}{3 + h} - \dfrac{1}{3}}{h}$$

e)

$$\lim_{x \to 4} f(x) \quad where \ f(x) = \begin{cases} \dfrac{x^2 - 16}{x - 4} & if \quad x \ne 4 \\ 3 & if \quad x = 4 \end{cases}$$

What is the type of discontinuity this function has at $x = 4$?

f)

$$\lim_{x \to 1} \arccos\left(\frac{\sqrt{x} - 2}{1 + \sqrt{x}}\right)$$