MAC 2311 TEST 3A
SPRING 2009

A. Sign your scantron sheet in the white area on the back <u>in ink</u>.

B. Write <u>and code</u> in the spaces indicated:

1) Name (last name, first initial, middle initial)
2) UF ID number
3) Discussion section number

C. Under "special codes", code in the test ID number 3, 1.

1 2 • 4 5 6 7 8 9 0
• 2 3 4 5 6 7 8 9 0

D. At the top right of your answer sheet, for "Test Form Code" encode A.

• B C D E

E. This test consists of 11 five-point multiple choice questions, 5 one-point bonus questions, and two sheets (4 pages) of partial credit questions worth 25 points. The time allowed is 90 minutes.

F. WHEN YOU ARE FINISHED:

1) Before turning in your test check for <u>transcribing errors</u>. Any mistakes you leave in are there to stay.

2) You must turn in your scantron and tear off sheets to your discussion leader. Be prepared to show your picture <u>I.D. with a legible signature</u>.

3) The answers will be posted on the MAC2311 homepage after the exam.

NOTE: Be sure to bubble the answers to questions 1–16 on your scantron.

Problems 1 - 11 are worth 5 points each.

1. The graph of $f(x) = (x^3 - 6x^2)^3$ is increasing on which of the following intervals?

   a. $(0, 4) \cup (6, \infty)$ b. $(-\infty, 4) \cup (6, \infty)$ c. $(0, 4)$ only

   d. $(-\infty, 0) \cup (4, 6)$ only e. $(-\infty, 0) \cup (4, \infty)$

---

2. Use L'Hospital's Rule to evaluate $\lim_{x \to 0} \dfrac{e^{2x} - 2e^x + 1}{\cos x - 1}$.

   a. -2 b. 0 c. 2 d. 1

   e. The limit does not exist.

---

3. The function $f(x) = \dfrac{x}{x^2 + 4}$ is both increasing and concave down on which of the following intervals?

   Note that $f'(x) = \dfrac{4 - x^2}{(x^2 + 4)^2}$ and $f''(x) = \dfrac{2x(x^2 - 12)}{(x^2 + 4)^3}$.

   a. $(-\infty, -2\sqrt{3}) \cup (0, 2)$

   b. $(-2\sqrt{3}, 0) \cup (2, \infty)$

   c. $(0, 2)$ only

   d. $(-2\sqrt{3}, 2\sqrt{3})$

   e. $(-\infty, -2) \cup (2\sqrt{3}, \infty)$

4. Find all values of c guaranteed by the Mean Value Theorem for $f(x) = x^3 + x$ on $[0, 3]$.

a. $c = 3$ only b. $c = \sqrt{3}$ only c. $c = 1$ only

d. $c = -\sqrt{3}$ and $c = \sqrt{3}$ e. $c = -3$ and $c = 3$

---

5. Find each value of x at which $f(x) = \dfrac{x^2 + 4}{x}$ has local (relative) extrema.

	local maximum(a) at $x =$_____	local minimum(a) at $x =$_____
a.	none	-2, 2
b.	0, 2	-2
c.	-2	2
d.	-2, 2	none
e.	2	-2

---

6. The edge of a cube was measured to be $x = 6$ inches. If the measurement of x has a possible error of ± 0.01 inch, use differentials to approximate the **percentage** error that could have occurred in using this measurement to compute the volume of the cube.

a. $\pm 2.16\%$ b. $\pm 0.6\%$ c. $\pm 1.08\%$ d. $\pm 0.5\%$ e. $\pm 0.75\%$

7. Find the absolute maximum and minimum values of $f(x) = \sin^2 x + \cos x$ on $\left[0, \frac{3\pi}{2}\right]$.

a. $\frac{5}{4}$ and 1 b. $\frac{5}{4}$ and -1 c. 1 and 0

d. 1 and -1 e. $\frac{5}{4}$ and 0

---

8. A rock thrown vertically upward from a lunar module 2.5 meters high with a velocity of 16 meters per second reaches a height of $h(t) = 2.5 + 16t - 0.8t^2$ meters in t seconds. Find the maximum height above the lunar surface reached by the rock, and its velocity when it hits the canopy of the module, also 2.5 meters high, on its way down.

a. maximum height: 82.5 meters velocity: -16 m/sec

b. maximum height: 80 meters velocity: -18.5 m/sec

c. maximum height: 82.5 meters velocity: 0 m/sec

d. maximum height: 112.5 meters velocity: -18.5 m/sec

e. maximum height: 112.5 meters velocity: -16 m/sec

---

9. **Approximate** the value of $\sqrt{3.4}$ by using the differential of $f(x) = \sqrt{x}$ at $x = 4$.

a. 1.725 b. 1.84 c. 1.826 d. 1.75 e. 1.85

10. Let $y = f(x)$ be a continuous function so that $f'(0) = f'(2) = 0$ and $f''(x) = 4x^3 - 6x^2$. Which of the following statements is/are true?

A. According to the Second Derivative Test, $f(x)$ has a local minimum at $x = 2$.

B. $f(x)$ is concave upward on $\left(\frac{3}{2}, \infty\right)$ only.

C. $f(x)$ has both a horizontal tangent line and an inflection point at $x = 0$.

a. A only
b. C only
c. B and C only
d. A and B only
e. A, B, and C

---

11. A ladder which is 25 feet long is leaning against a wall and makes an angle θ with the ground. If the base of the ladder is being pulled away from the wall at the rate of 2 feet per minute, how fast is θ changing when the base of the ladder is 20 feet from the wall?

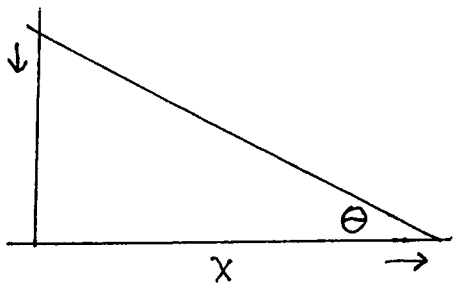
a. $-\frac{2}{15}$ radians per minute

b. $-\frac{1}{5}$ radians per minute

c. $-\frac{1}{10}$ radians per minute

d. $-\frac{3}{15}$ radians per minute

e. $-\frac{3}{10}$ radians per minute

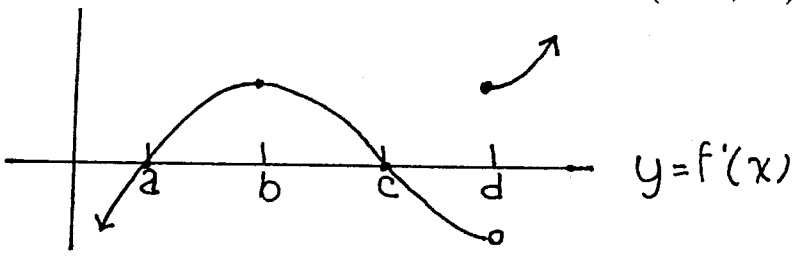


Be sure to work the bonus problems on the next page!

Bonus! (1 point each) Bubble (a) for True or (b) for False.

Given the graph of the **derivative** $f'(x)$, which of the following statements is/are true of the graph of $y = f(x)$?

Assume that $f(x)$ is continuous with domain $(-\infty, \infty)$.



12. $f(x)$ is decreasing on (b, d).

    a. True b. False

---

13. $f(x)$ is concave upwards on the intervals $(-\infty, b) \cup (d, \infty)$.

    a. True b. False

---

14. $f(x)$ has a local minimum at $x = d$.

    a. True b. False

---

15. $f(x)$ has an inflection point at $x = b$.

    a. True b. False

---

16. Rolle's Theorem can be applied to the function $f'(x)$ on $[a, c]$.

    Assume that $f'(x)$ is differentiable on (a, d).

    a. True b. False

MAC 2311 Test 3A Part II

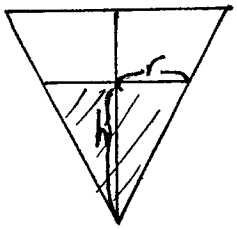
Spring 2009

Sect#_____ Name_____

U.F. ID_____ Signature_____

SHOW ALL WORK TO RECEIVE FULL CREDIT.

1. A container in the shape of an inverted cone has diameter 8 inches and height of 12 inches. The container is being filled with liquid at the rate of 3π cubic inches per minute.



a) Express r in terms of h.

$r =$_____

b) How fast is the height of the liquid rising when the liquid is 6 inches high in the container? Be sure to include units in your answer.

Hint: $V = \frac{1}{3}\pi r^2 h$.

$\frac{dh}{dt} =$_____

2. If $f(x) = (x-2)^{\frac{2}{3}}(x+3), f'(x) = \dfrac{5x}{3(x-2)^{\frac{1}{3}}}, f''(x) = \dfrac{10(x-3)}{9(x-2)^{\frac{4}{3}}}$ and $f(0) \approx 4.8$.

A. complete the following (if none, write "none").

1) domain of f _____

2) critical number(s) at $x =$ _____

3) horizontal tangent line at $x =$ _____

vertical tangent line or cusp at $x =$_____

4) number lines for f' and f''

_____ _____

5) relative maximum at $x =$_____ maximum value:_____

relative minimum at $x =$_____ minimum value: _____

6) inflections point(s) (x, y): _____

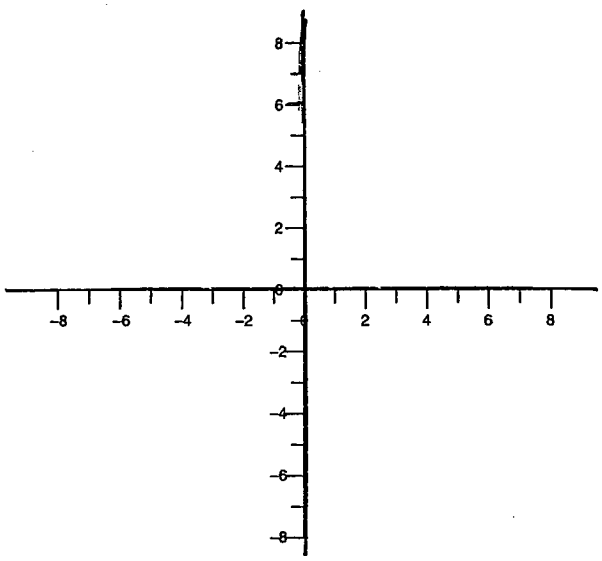
7) intercept(s): $x =$_____

$y =$_____

Sect#_____ Name_____

2. (continued)

B. Sketch the graph of $y = (x-2)^{\frac{2}{3}}(x+3)$, labeling all significant points from part (A). Note that $f(0) \approx 4.8$.



3. The position (in feet) of an object moving along a straight track at time t (in seconds) is given by $s(t) = 2t^3 - 9t^2 + 12t$ for $t \geq 0$.

a) When is the particle moving in a negative direction?

_____ $\leq t \leq$ _____ sec.

b) What is the total distance traveled from $t = 0$ to $t = 4$ seconds?

_____ feet

4. Let $f(x) = x \ln x$.

a) Find the linearization of $f(x)$ at $x = e$.

$L(x) =$_____

b) Use the linearization from (a) to approximate $3 \ln 3$. Leave your answer in terms of e.

$3 \ln 3 \approx$_____

c) Find the relative extreme values of $f(x) = x \ln x$. If none, write "none". Be sure to consider domain.

relative maximum(a)_____

relative minimum (a)_____

MAC 2311 TEST 3B
SPRING 2009

A. Sign your scantron sheet in the white area on the back in ink.

B. Write and code in the spaces indicated:

1) Name (last name, first initial, middle initial)
2) UF ID number
3) Discussion section number

C. Under "special codes", code in the test ID number 3, 2.

1	2	•	4	5	6	7	8	9	0
1	•	3	4	5	6	7	8	9	0

D. At the top right of your answer sheet, for "Test Form Code" encode B.

A	•	C	D	E

E. This test consists of 11 five-point multiple choice questions, 5 one-point bonus questions, and two sheets (4 pages) of partial credit questions worth 25 points. The time allowed is 90 minutes.

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3) The answers will be posted on the MAC2311 homepage after the exam.

NOTE: Be sure to bubble the answers to questions 1–16 on your scantron.

Problems 1 - 11 are worth 5 points each.

1. The graph of $f(x) = (x^3 - 9x^2)^3$ is increasing on which of the following intervals?

   a. $(0, 6)$ only b. $(-\infty, 0) \cup (6, \infty)$ c. $(0, 6) \cup (9, \infty)$

   d. $(-\infty, 6) \cup (9, \infty)$ e. $(-\infty, 0) \cup (6, 9)$ only

---

2. The function $f(x) = \dfrac{x}{x^2 + 4}$ is both increasing and concave down on which of the following intervals?

   Note that $f'(x) = \dfrac{4 - x^2}{(x^2 + 4)^2}$ and $f''(x) = \dfrac{2x(x^2 - 12)}{(x^2 + 4)^3}$.

   a. $(-2\sqrt{3}, 0) \cup (2, \infty)$

   b. $(-\infty, -2) \cup (2\sqrt{3}, \infty)$

   c. $(0, 2)$ only

   d. $(-\infty, -2\sqrt{3}) \cup (0, 2)$

   e. $(-2\sqrt{3}, 2\sqrt{3})$

---

3. Find all values of c guaranteed by the Mean Value Theorem for $f(x) = x^3 + x$ on $[0, 3]$.

   a. $c = \sqrt{3}$ only b. $c = 1$ only c. $c = 3$ only

   d. $c = -3$ and $c = 3$ e. $c = -\sqrt{3}$ and $c = \sqrt{3}$

4. Use L'Hospital's Rule to evaluate $\lim_{x \to 0} \dfrac{e^{3x} - 3e^x + 2}{\cos x - 1}$.

a. 2 b. 6 c. 0 d. -6

e. The limit does not exist.

---

5. Find each value of x at which $f(x) = \dfrac{x^2 + 9}{x}$ has local (relative) extrema.

	local maximum(a) at $x =$_____	local minimum(a) at $x =$_____
a.	3	-3
b.	-3, 3	none
c.	0, 3	-3
d.	none	-3, 3
e.	-3	3

---

6. **Approximate** the value of $\sqrt{3.4}$ by using the differential of $f(x) = \sqrt{x}$ at $x = 4$.

a. 1.84 b. 1.725 c. 1.85 d. 1.826 e. 1.75

7. Find the absolute maximum and minimum values of $f(x) = \sin^2 x + \cos x$ on $\left[0, \frac{3\pi}{2}\right]$.

a. $\frac{5}{4}$ and 0 b. 1 and -1 c. $\frac{5}{4}$ and -1

d. 1 and 0 e. $\frac{5}{4}$ and 1

---

8. A rock thrown vertically upward from a lunar module 2.6 meters high with a velocity of 16 meters per second reaches a height of $h(t) = 2.6 + 16t - 0.8t^2$ meters in t seconds. Find the maximum height above the lunar surface reached by the rock, and its velocity when it hits the canopy of the module, also 2.6 meters high, on its way down.

a. maximum height: 112.6 meters velocity: -18.6 m/sec

b. maximum height: 82.6 meters velocity: -16 m/sec

c. maximum height: 112.6 meters velocity: -16 m/sec

d. maximum height: 82.6 meters velocity: 0 m/sec

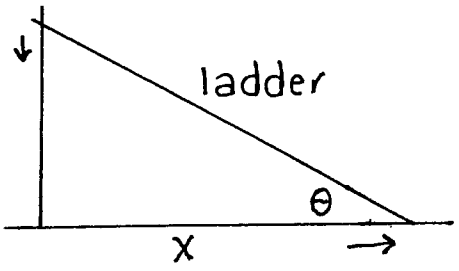
e. maximum height: 80 meters velocity: -18.6 m/sec

---

9. The edge of a cube was measured to be $x = 6$ inches. If the measurement of x has a possible error of ± 0.01 inch, use differentials to approximate the **percentage** error that could have occurred in using this measurement to compute the volume of the cube.

a. $\pm 0.75\%$ b. $\pm 2.16\%$ c. $\pm 0.6\%$ d. $\pm 0.5\%$ e. $\pm 1.08\%$

10. A ladder which is 25 feet long is leaning against a wall and makes an angle θ with the ground. If the base of the ladder is being pulled away from the wall at the rate of 2 feet per minute, how fast is θ changing when the base of the ladder is 20 feet from the wall?



a. $-\frac{3}{10}$ radians per minute

b. $-\frac{2}{15}$ radians per minute

c. $-\frac{3}{15}$ radians per minute

d. $-\frac{1}{10}$ radians per minute

e. $-\frac{1}{5}$ radians per minute

---

11. Let $y = f(x)$ be a continuous function so that $f'(0) = f'(2) = 0$ and $f''(x) = 4x^3 - 6x^2$. Which of the following statements is/are true?

A. According to the Second Derivative Test, $f(x)$ has a local minimum at $x = 2$.

B. $f(x)$ is concave upward on $\left(\frac{3}{2}, \infty\right)$ only.

C. $f(x)$ has both a horizontal tangent line and an inflection point at $x = 0$.

a. C only b. A only c. A, B and C

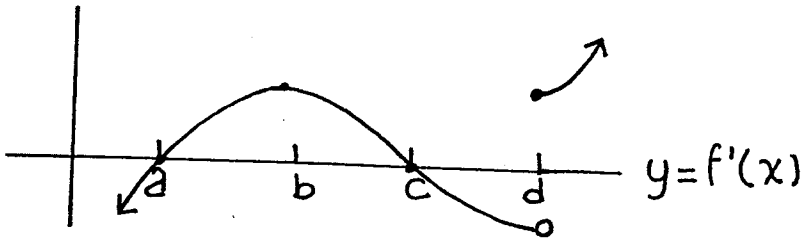
d. B and C only e. A and B only

Be sure to work the bonus problems on the next page!

Bonus! (1 point each) Bubble (a) for True or (b) for False.

Given the graph of the **derivative** $f'(x)$, which of the following statements is/are true of the graph of $y = f(x)$?

Assume that $f(x)$ is continuous with domain $(-\infty, \infty)$.



12. Rolle's Theorem can be applied to the function $f'(x)$ on $[a, c]$.
Assume that $f'(x)$ is differentiable on (a, d).

a. True b. False

---

13. $f(x)$ is decreasing on (b, d).

a. True b. False

---

14. $f(x)$ is concave upwards on the intervals $(-\infty, b) \cup (d, \infty)$.

a. True b. False

---

15. $f(x)$ has a local minimum at $x = d$.

a. True b. False

---

16. $f(x)$ has an inflection point at $x = b$.

a. True b. False

MAC 2311 Test 3B Part II

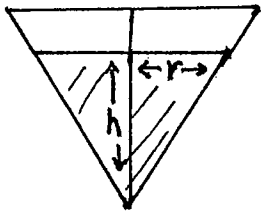
Spring 2009

Sect#_____ Name_____

U.F. ID_____ Signature_____

SHOW ALL WORK TO RECEIVE FULL CREDIT.

1. A container in the shape of an inverted cone has diameter 8 inches and height of 16 inches. The container is being filled with liquid at the rate of 5π cubic inches per minute.



a) Express r in terms of h.

$r =$_____

b) How fast is the height of the liquid rising when the liquid is 8 inches high in the container? Be sure to include units in your answer.

Hint: $V = \frac{1}{3}\pi r^2 h$.

$\frac{dh}{dt} =$_____

2. If $f(x) = (x+2)^{\frac{2}{3}}(x-3), f'(x) = \dfrac{5x}{3(x+2)^{\frac{1}{3}}}, f''(x) = \dfrac{10(x+3)}{9(x+2)^{\frac{4}{3}}}$ and $f(0) \approx -4.8$.

A. complete the following (if none, write "none").

1) domain of f _____

2) critical number(s) at $x =$ _____

3) horizontal tangent line at $x =$ _____

vertical tangent line or cusp at $x =$_____

4) number lines for f' and f''

_____ _____

5) relative maximum at $x =$_____ maximum value:_____

relative minimum at $x =$_____ minimum value: _____

6) inflections point(s) (x, y): _____

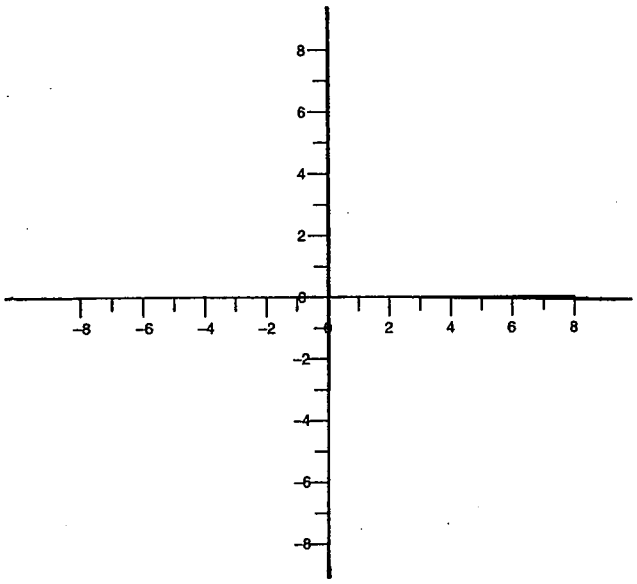
7) intercept(s): $x =$_____

$y =$_____

Sect#_____ Name_____

2. (continued)

B. Sketch the graph of $y = (x+2)^{\frac{2}{3}}(x-3)$, labeling all significant points from part (A). Note that $f(0) \approx -4.8$.



3. The position (in feet) of an object moving along a straight track at time t (in seconds) is given by $s(t) = 2t^3 - 9t^2 + 12t$ for $t \geq 0$.

a) When is the particle moving in a negative direction?

_____ $\leq t \leq$ _____ sec.

b) What is the total distance traveled from $t = 0$ to $t = 5$ seconds?

_____ feet

4. Let $f(x) = x \ln x$.

a) Find the linearization of $f(x)$ at $x = e$.

$L(x) =$_____

b) Use the linearization from (a) to approximate $2 \ln 2$. Leave your answer in terms of e.

$2 \ln 2 \approx$_____

c) Find the relative extreme values of $f(x) = x \ln x$. If none, write "none". Be sure to consider domain.

relative minimum (a)_____

relative maximum (a)_____