MAC 2311 TEST 2A
SPRING 2009

A. Sign your scantron sheet in the white area on the back in ink.

B. Write and code in the spaces indicated:

1) Name (last name, first initial, middle initial)
2) UF ID number
3) Discussion section number

C. Under "special codes", code in the test ID number 2, 1.

1	•	3	4	5	6	7	8	9	0
•	2	3	4	5	6	7	8	9	0

D. At the top right of your answer sheet, for "Test Form Code" encode A.

• B C D E

E. This test consists of 5 three-point and 8 five-point multiple choice questions, 4 bonus questions, and two sheets (4 pages) of partial credit questions worth 25 points. The time allowed is 90 minutes.

F. WHEN YOU ARE FINISHED:

1) Before turning in your test check for transcribing errors. Any mistakes you leave in are there to stay.

2) You must turn in your scantron and tear off sheets to your discussion leader. Be prepared to show your picture I.D. with a legible signature.

3) The answers will be posted on the MAC2311 homepage after the exam.

Questions 1 - 5 are worth 3 points each.

1. If $f(x) = \dfrac{x^{\frac{3}{2}} - x + 1}{x}$, then $f'(x) =$____________.

a. $\sqrt{x} - 1 + \dfrac{1}{x}$ b. $\dfrac{\sqrt{x}+1}{x}$ c. $\dfrac{x^{\frac{3}{2}} - 2}{2x^2}$

d. $\dfrac{1}{2\sqrt{x}} - 1 - \dfrac{1}{x^2}$ e. $\dfrac{3\sqrt{x} - 2}{2}$

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2. Find the derivative of $f(x) = \sec^2(3x^2)$.

(a) $f'(x) = 12x\sec(3x^2)$

(b) $f'(x) = 2\sec^2(3x^2)\tan(3x^2)$

(c) $f'(x) = 6x\sec(6x)\tan(6x)$

(d) $f'(x) = 12x\sec^2(3x^2)\tan(3x^2)$

(e) $f'(x) = 12x\sec(3x^2)\tan^2(3x^2)$

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3. Use the definition of derivative to evaluate $\lim\limits_{h\to 0} \dfrac{(x+h)^{\frac{1}{5}} - x^{\frac{1}{5}}}{h}$.

a. $\dfrac{1}{5x^{\frac{4}{5}}}$ b. $\dfrac{1}{5x^4}$ c. $\dfrac{x^{\frac{4}{5}}}{5}$ d. $\dfrac{5}{x^{\frac{1}{5}}}$ e. The limit does not exist.

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4. Find $f'(0)$ if $f(x) = 4^x + e^{2\tan x}$.

a. $\ln 4 + 1$ b. 3 c. $\ln 4 + 2$ d. 2 e. $\ln 4 + 2e$

5. A pollutant from a factory is carried away by wind currents. Its concentration in the air is given by $P(x) = \dfrac{0.4}{3x - 2}$ where $P(x)$ is measured in parts per million and x is the pollutant's distance in miles from the factory. Find the rate at which the concentration of the pollutant is changing (the instantaneous rate of change of P with respect to x) when the pollutant is four miles from the factory.

a. The concentration is decreasing by 0.4 parts/million.

b. The concentration is decreasing by 1.2 parts per million.

c. The concentration is increasing by 0.4 parts per million.

d. The concentration is increasing by 0.012 parts per million.

e. The concentration is decreasing by 0.012 parts per million.

Problems 6 - 13 are worth 5 points each.

6. If $f(x) = \dfrac{\sin(\pi x)}{6x}$, let $p = \lim_{x \to 0} f(x)$ and let $q = \lim_{x \to \infty} f(x)$. Find p and q.

a. $p = \dfrac{1}{6}$ and $q = 1$

b. $p = \dfrac{\pi}{6}$ and $q = 0$

c. $p = \dfrac{1}{6\pi}$ and $q = 0$

d. $p = \dfrac{\pi}{6}$ and $q = 1$

e. $p = \dfrac{1}{6\pi}$ and $q = \infty$

7. Write the equation of the normal line to the curve $y = \sqrt{5 - e^{3x}}$ at $x = 0$.

a. $y = \frac{4}{3}x + \frac{2}{3}$ b. $y = -4x + 2$ c. $y = \frac{4}{3}x + 2$

d. $y = -4x - \frac{1}{2}$ e. $y = -\frac{3}{4}x + 2$

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8. Find each point at which the tangent line to the curve $y = 2x + \frac{4}{x} + 1$ is parallel to the line $y + 2x = 6$.

a. $(1, 7)$ and $(-1, -5)$

b. $(1, 4)$ and $(-2, 10)$

c. $(1, 7)$ and $(-2, -5)$

d. $(1, 4)$ and $(-1, 8)$

e. There are no such points.

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9. Find each value of x at which $f(x) = x(2x - 8)^3$ has a horizontal tangent line.

a. $x = 0$ and $x = 4$ only

b. $x = 4$ and $x = \frac{8}{5}$

c. $x = 0$, $x = 1$ and $x = 4$

d. $x = 0$ and $x = 2$

e. $x = 1$ and $x = 4$ only

10. Find $f'(3)$ if $f(x) = \dfrac{2x}{\sqrt{x^2 - 4x + 7}}$.

a. $-\dfrac{5}{8}$ b. $\dfrac{1}{4}$ c. $-\dfrac{1}{2}$ d. $\dfrac{5}{8}$ e. $-\dfrac{1}{4}$

---

11. Find $\dfrac{dy}{dx}$ if $3xy - y^2 = 4 + x^2$. $\dfrac{dy}{dx} =$ __________.

a. $\dfrac{2x - y + 4}{3x}$ b. $\dfrac{3x + 2y}{2x - 3y}$ c. $\dfrac{2x}{3 - 2y}$

d. $\dfrac{2x - 3y}{3x - 2y}$ e. $\dfrac{4 + 2x - 3y}{3x - 2y}$

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12. Given functions f and g so that $f(4) = -3$, $f'(4) = 6$, $f'(-3) = 2$, $g(-3) = \dfrac{1}{2}$, $g'(-3) = -\dfrac{2}{3}$, $g(4) = 4$ and $g'(4) = \dfrac{1}{4}$.
If $h(x) = (g \circ f)(x)$, find $h'(4)$.

a. -4 b. $-\dfrac{2}{3}$ c. -1 d. 3 e. $\dfrac{3}{2}$

---

13. Use logarithmic differentiation to find the slope of the curve $y = \dfrac{e^{2x}(x + 4)}{\sqrt{2x + 1}}$ at $x = 0$.

a. 7 b. -3 c. 5 d. $\dfrac{5}{4}$ e. 0

Be sure to work the bonus problems on the next page!

Bonus!!

Problems 14 - 16 are 1 point each.

Bubble (a) for True or (b) for False.

14. $\frac{d}{dx}\left(x^e\right) = ex^{e-1}$.

a. True b. False

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15. $\frac{d}{dx}(\ln 4) = \frac{1}{4}$.

a. True b. False

---

16. The function $f(x) = x^{\frac{2}{3}}$ has a horizontal tangent line at $x = 0$.

a. True b. False

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17. **(2 points)** If $f(x) = \sin^{-1} x$, then $f''(x) =$____________.

a. $\frac{-\cos x}{\sin^2 x}$ b. $\frac{-1}{2(1-x^2)^{\frac{3}{2}}}$ c. $\frac{-2x}{1-x^2}$ d. $\frac{x}{(1-x^2)^{\frac{3}{2}}}$

MAC 2311 Test 2A, Part II
Spring 2009

Sect # ______________ Name ______________________________

UF ID ______________ Signature ______________________________

SHOW ALL WORK TO RECEIVE FULL CREDIT.

1. a) Use the limit definition of derivative to find $f'(x)$ if $f(x) = \dfrac{x}{x+1}$.

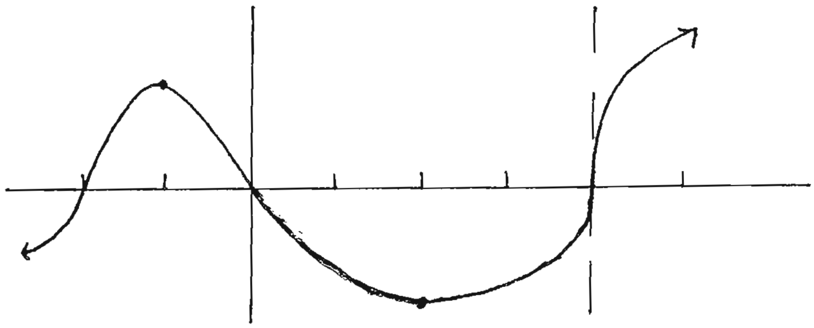
b) For what x-values is $f(x)$ differentiable? Write your answer in interval notation.

____

c) Write the equation of the tangent line to $f(x)$ at $x = 1$.

$y =$______________________

2. The sketch of a function $f(x)$ is given below. Use the graph to find the following information about the **derivative** $f'(x)$.

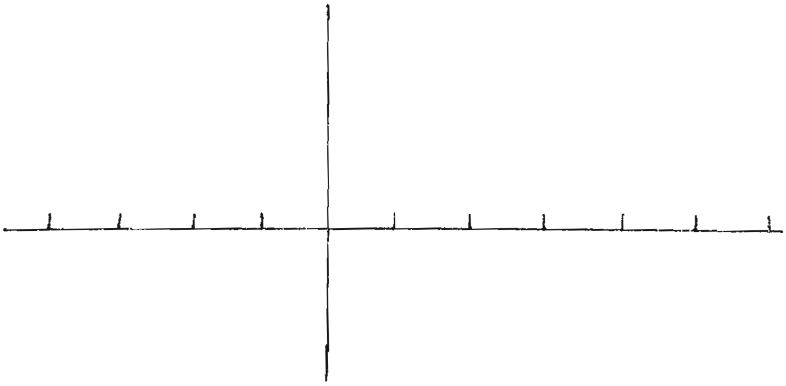


(a) $f'(0)$ ________ 0 (insert the symbol $<$, $=$, or $>$)

(b) $f(x)$ is not differentiable at $x =$ ____________.

(c) $f'(x) = 0$ at $x =$ ____________.

(d) Use the above information to sketch a possible graph of $f'(x)$.



3. Evaluate $\lim_{x \to 0} \dfrac{1 - \sec x}{1 - \cos x}$.

Sect # ______________ Name ______________________________

4. Let $f(x) = \begin{cases} 2\sin x & x \leq 0 \\ kx & x > 0 \end{cases}$.

(a) Find $f(0)$.

(b) Use the definition of continuity to show that $f(x)$ is continuous at $x = 0$.

(c) Use the limit definition $f'(0) = \lim_{x \to 0} \frac{f(x) - f(0)}{x - 0}$ to find the value of k that will make $f(x)$ differentiable at $x = 0$.

$k =$ ____________

5. Let $f(x) = x^{x/2}$.

(a) Use logarithmic differentiation to find $f'(x)$.

$f'(x) =$ ______________________

(b) Find the slope of the tangent line to $f(x)$ at $x = 2$. Leave your answer in terms of ln.

$m =$ ______________________

6. Find the derivative of the function $g(x) = \log_4(2 + \cos x) + \tan^{-1}(4x)$.

MAC 2311 Test 2B
Spring 2009

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B. Write and code in the spaces indicated:

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2) UF ID number
3) Discussion section number

C. Under "special codes", code in the test ID number 2, 2.

1 • 3 4 5 6 7 8 9 0

1 • 3 4 5 6 7 8 9 0

D. At the top right of your answer sheet, for "Test Form Code" encode B.

A • C D E

E. This test consists of 5 three-point and 8 five-point multiple choice questions, 4 bonus questions, and two sheets (4 pages) of partial credit questions worth 25 points. The time allowed is 90 minutes.

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3) The answers will be posted on the MAC2311 homepage after the exam.

Questions 1 - 5 are worth 3 points each.

1. If $f(x) = \dfrac{x^{\frac{3}{2}} - 3x + 1}{x}$, then $f'(x) =$_____________.

a. $\dfrac{\sqrt{x}+1}{x}$ b. $\sqrt{x} - 3 + \dfrac{1}{x}$ c. $\dfrac{1}{2\sqrt{x}} - 3 - \dfrac{1}{x^2}$

d. $\dfrac{3\sqrt{x}-2}{2}$ e. $\dfrac{x^{\frac{3}{2}} - 2}{2x^2}$

---

2. Find the derivative of $f(x) = \sec^2(4x^2)$.

(a) $f'(x) = 2\sec^2(4x^2)\tan(4x^2)$

(b) $f'(x) = 8x\sec(8x)\tan(8x)$

(c) $f'(x) = 16x\sec^2(4x^2)\tan(4x^2)$

(d) $f'(x) = 16x\sec(4x^2)\tan^2(4x^2)$

(e) $f'(x) = 16x\sec(4x^2)$

---

3. Use the definition of derivative to evaluate $\lim\limits_{h\to 0} \dfrac{(x+h)^{\frac{1}{4}} - x^{\frac{1}{4}}}{h}$.

a. The limit does not exist. b. $\dfrac{1}{4x^{\frac{3}{4}}}$ c. $\dfrac{x^{\frac{3}{4}}}{4}$ d. $\dfrac{4}{x^{\frac{1}{4}}}$ e. $\dfrac{1}{4x^3}$

---

4. Find $f'(0)$ if $f(x) = 3^x + e^{4\tan x}$.

a. 2 b. $\ln 3 + 4e$ c. 5 d. $\ln 3 + 4$ e. $\ln 3 + 1$

5. A pollutant from a factory is carried away by wind currents. Its concentration in the air is given by $P(x) = \dfrac{0.5}{3x-2}$ where $P(x)$ is measured in parts per million and x is the pollutant's distance in miles from the factory. Find the rate at which the concentration of the pollutant is changing (the instantaneous rate of change of P with respect to x) when the pollutant is four miles from the factory.

(a) The concentration is decreasing by 0.5 parts/million.

(b) The concentration is increasing by 0.015 parts per million.

(c) The concentration is decreasing by 0.015 parts per million.

(d) The concentration is increasing by 0.5 parts per million.

(e) The concentration is decreasing by 1.5 parts per million.

Problems 6 - 13 are worth 5 points each.

6. If $f(x) = \dfrac{\sin(\pi x)}{4x}$, let $p = \lim_{x\to 0} f(x)$ and let $q = \lim_{x\to\infty} f(x)$. Find p and q.

(a) $p = \dfrac{\pi}{4}$ and $q = 0$

(b) $p = \dfrac{1}{4\pi}$ and $q = \infty$

(c) $p = \dfrac{1}{4}$ and $q = 1$

(d) $p = \dfrac{1}{4\pi}$ and $q = 0$

(e) $p = \dfrac{\pi}{4}$ and $q = 1$

7. Find each point at which the tangent line to the curve $y = 2x + \frac{4}{x} + 1$ is parallel to the line $y + 2x = 6$.

(a) $(1, 7)$ and $(-2, -5)$

(b) $(1, 4)$ and $(-1, 8)$

(c) $(1, 4)$ and $(-2, 10)$

(d) $(1, 7)$ and $(-1, -5)$

(e) There are no such points.

---

8. Write the equation of the normal line to the curve $y = \sqrt{5 - e^{3x}}$ at $x = 0$.

a. $y = -\frac{3}{4}x + 2$ b. $y = -4x - \frac{1}{2}$ c. $y = \frac{4}{3}x + \frac{2}{3}$

d. $y = -4x + 2$ e. $y = \frac{4}{3}x + 2$

---

9. Find each value of x at which $f(x) = x(4x-8)^3$ has a horizontal tangent line.

a. $x = 2$ and $x = \frac{8}{7}$

b. $x = 0$ and $x = 2$ only

c. $x = 0$, $x = 2$ and $x = \frac{1}{2}$

d. $x = 2$ and $x = \frac{1}{2}$ only

e. $x = 0$ and $x = 4$

10. Find $f'(3)$ if $f(x) = \dfrac{2x}{\sqrt{x^2 - 4x + 7}}$.

a. $-\dfrac{1}{4}$ b. $\dfrac{5}{8}$ c. $\dfrac{1}{4}$ d. $-\dfrac{5}{8}$ e. $-\dfrac{1}{2}$

---

11. Given functions f and g so that $f(4) = -3$, $f'(4) = 6$, $f'(-3) = 2$,
$g(-3) = \dfrac{1}{2}$, $g'(-3) = -\dfrac{2}{3}$, $g(4) = 4$ and $g'(4) = \dfrac{1}{4}$.
If $h(x) = (g \circ f)(x)$, find $h'(4)$.

a. -4 b. -1 c. $\dfrac{3}{2}$ d. $-\dfrac{2}{3}$ e. 3

---

12. Use logarithmic differentiation to find the slope of the curve $y = \dfrac{e^{2x}(x+4)}{\sqrt{2x+1}}$ at $x = 0$.

a. 0 b. $\dfrac{5}{4}$ c. 7 d. -3 e. 5

---

13. Find $\dfrac{dy}{dx}$ if $3xy - y^2 = 6 + x^2$. $\dfrac{dy}{dx} =$ ____________.

a. $\dfrac{6 + 2x - 3y}{3x - 2y}$ b. $\dfrac{2x - 3y}{3x - 2y}$ c. $\dfrac{2x}{3 - 2y}$

d. $\dfrac{2x - y + 6}{3x}$ e. $\dfrac{3x + 2y}{2x - 3y}$

Be sure to work the bonus problems on the next page!

Bonus!!

Problems 14 - 16 are 1 point each.

Bubble (a) for True or (b) for False.

14. $\frac{d}{dx}(\ln 3) = \frac{1}{3}$.

a. True   b. False

---

15. $\frac{d}{dx}(x^e) = ex^{e-1}$.

a. True   b. False

---

16. The function $f(x) = x^{\frac{1}{3}}$ has a horizontal tangent line at $x = 0$.

a. True   b. False

---

17. **(2 points)** If $f(x) = \sin^{-1} x$, then $f''(x) =$____________.

a. $\frac{-1}{2(1-x^2)^{\frac{3}{2}}}$ b. $\frac{-2x}{1-x^2}$ c. $\frac{x}{(1-x^2)^{\frac{3}{2}}}$ d. $\frac{-\cos x}{\sin^2 x}$

MAC 2311 Test 2B, Part II
Spring 2009

Sect # ______________ Name ______________________________

UF ID ______________ Signature ______________________________

SHOW ALL WORK TO RECEIVE FULL CREDIT.

1. a) Use the limit definition of derivative to find $f'(x)$ if $f(x) = \dfrac{x}{x+2}$.

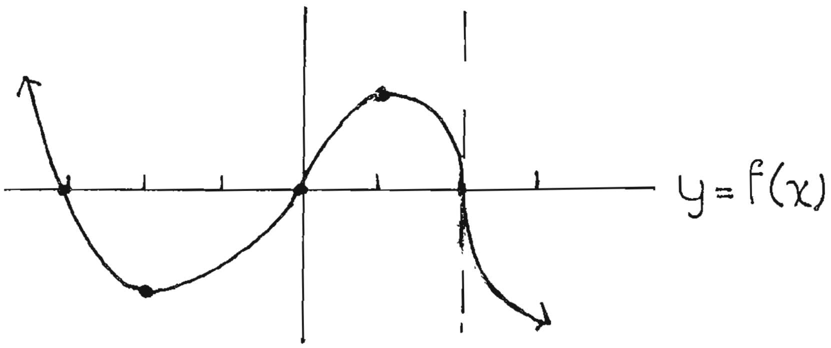
b) For what x-values is $f(x)$ differentiable? Write your answer in interval notation.

____

c) Write the equation of the tangent line to $f(x)$ at $x = 2$.

$y =$____________________

2. The sketch of a function $f(x)$ is given below. Use the graph to find the following information about the **derivative** $f'(x)$.

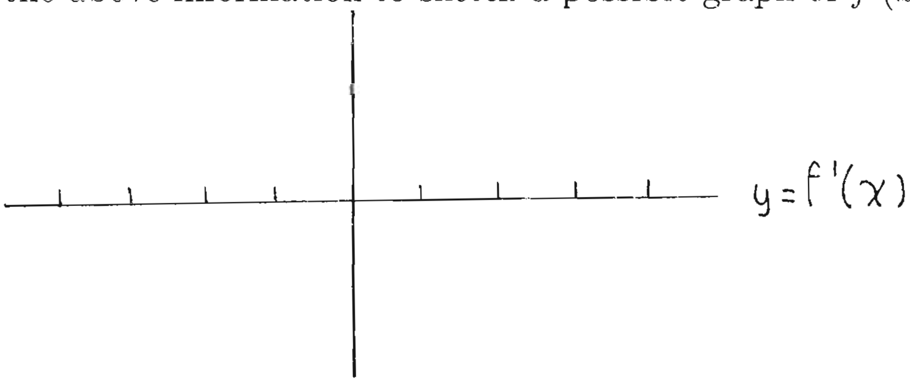


(a) $f'(0)$ ________ 0 (insert the symbol $<$, $=$, or $>$)

(b) $f(x)$ is not differentiable at $x =$____________.

(c) $f'(x) = 0$ at $x =$____________.

(d) Use the above information to sketch a possible graph of $f'(x)$.



3. Evaluate $\lim_{x \to 0} \dfrac{\sec x - 1}{1 - \cos x}$.

Sect # ________________ Name ________________________________

4. Let $f(x) = \begin{cases} 4\sin x & x \leq 0 \\ kx & x > 0 \end{cases}$.

(a) Find $f(0)$.

(b) Use the definition of continuity to show that $f(x)$ is continuous at $x = 0$.

(c) Use the limit definition $f'(0) = \lim_{x \to 0} \dfrac{f(x) - f(0)}{x - 0}$ to find the value of k that will make $f(x)$ differentiable at $x = 0$.

$k =$ ____________

5. Find the derivative of the function $g(x) = \log_2(3 + \cos x) + \tan^{-1}(3x)$.

6. Let $f(x) = x^{x/3}$.

(a) Use logarithmic differentiation to find $f'(x)$.

$f'(x) =$ ____________________

(b) Find the slope of the tangent line to $f(x)$ at $x = 3$. Leave your answer in terms of ln.

$m =$ ____________________